SIMILARITY OF CONDENSATION PROCESSES IN EXPANDING CO_2 JETS

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Numerous investigations of condensation processes in supersonic jets have shown that when certain relationships connecting the nozzle geometry and the stagnation pressure and temperature are satisfied, these processes take place in a similar way. The condition under which the growth of clusters takes place in a similar way and results in the same average cluster size was obtained in [1] for an ideal gas:

$$p_0 T_0^{(1.25\gamma-0.5)/(1-\gamma)} = A_d. \tag{1}$$

Here p_0 and T_0 are the stagnation pressure and temperature, respectively; γ is the ratio of heat capacities; A_d is a quantity which depends on the nozzle geometry. This relation describes monatomic gases and nitrogen well. In the expansion of polyatomic gases with condensation, when the pressures are not so low that vibrational relaxation is frozen in, the heat capacity becomes a function of temperature. This has a pronounced influence on the distribution of gasdynamic parameters in the stream [2] and on the position of the point where the saturated state is reached. The set of stagnation parameters $\ln p_0$, $\ln T_0$ yielding the same average cluster size does not have a linear dependence [1].

Attempts to use a linear approximation [3, 4], possible for small temperature intervals, lead to the results being interpreted in terms of the "effective index of the isentrope of expansion," although it is known that the description of the expansion of a polyatomic gas by a constant ratio of heat capacities is conditional and limited [2, 5]. In such an approximation it is difficult to compare experimental results obtained in different temperature intervals.

The present work is devoted to a search for the conditions of similarity of processes of condensation for a gas whose heat capacity depends on temperature. To simplify the analysis we use a model of equilibrium spherical expansion of a gas into a vacuum, which well describes the far flow field of axisymmetric jets near the axis. The temperature dependence of the heat capacity of CO_2 is taken from [6].

The main arguments are made by analogy with those of [1]. We shall assume that the kinetics of the growth of a cluster with a size N and an area Q is determined by bimolecular reactions with the participation of monomers and monomolecular reactions of spontaneous evaporation of molecules from the surface of the cluster. Thus, we exclude from consideration the stage of formation of dimers, for which triple collisions are important [7]. Let us consider a certain section of expansion on the jet axis corresponding to the interval of temperature variation (T, T − dT). Then the influence of bimolecular reactions on the process of cluster growth will be characterized by the number of collisions of molecules with the cluster in this interval (dz) and the influence of spontaneous evaporation corresponding to the time interval dt. To determine the dependence of these quantities on the stagnation conditions we consider the spherical escape of gas into a vacuum. The initial system of equations of continuity, state, energy, and motion has the form

$$r^2 u\rho = \text{const}; \tag{2}$$

$$p = \rho RT; \tag{3}$$

$$u^2/2 + h = h_0; \tag{4}$$

$$\rho u du + dp = 0, \tag{5}$$

where r is the coordinate; u, velocity; ρ, density; p, pressure; h, specific enthalpy; R, gas constant. Here and later, stagnation parameters are denoted by the index 0 and parameters at the critical sphere by the index *. The solution of the system is determined if p_0, T_0, and

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r_* are assigned. From these equations (or directly from the thermodynamic expression for the internal-energy differential de = Tds − pdv) one can obtain the following condition of constancy of the entropy s for a gas with a variable heat capacity: $R \ln p = \int c_p \frac{dT}{T} + A_s$ or, designating $\int c_p \frac{dT}{T} \equiv J(T)$,

$$R \ln p = J(T) + A_s. \tag{6}$$

Here $c_p = c_p(T)$ is the heat capacity at constant pressure; A_S is a constant characterizing the given isentrope, analogous to the ratio of heat capacities for c_p = const.

In the case of variable heat capacity the expression for the enthalpy acquires the form $h = \int c_p dT$ + const, or, designating the integral as J_h. we obtain

$$h = J_h(T) + \text{const}. \tag{7}$$

The gas temperature T_* at the critical sphere is determined by the numerical solution of the equation $2[J_h(T_0) - J_h(T_*)] = RT_*/(1 - R/c_p(T_*))$, while the velocity u_* is determined from the energy equation (4); using Eq. (6), we write the pressure p_* in the form $p_* = p_0 \exp [(J_* - J_0)/R]$.

The time interval dt is defined by the expression

$$dt = dr/u. \tag{8}$$

We obtain the expression for dr by differentiating the continuity equation (2),

$$dr = -0.5r_* (\rho_* u_*/\rho u)^{0.5}(du/u + d\rho/\rho),$$

or, using Eqs. (3) and (5), we have

$$dr = dT 0.5r_* (\rho_* u_*/\rho u)^{0.5} c_p (1 - M^2)/u^2, \tag{9}$$

where M is the Mach number.

The frequency of collisions of molecules with the cluster is determined by the expression

$$f = 0.25(p/kT)Q(8kT/\pi m)^{0.5}. \tag{10}$$

With allowance (8)-(10) for the number of collisions of monomers with the cluster in the interval dT we have

$$dz = dT (8k\pi m)^{-0.5} Q r_* p c_p (1 - M^2)(\rho_* u_*/\rho u)^{0.5}/(u^3 T^{0.5}). \tag{11}$$

For jets with initial conditions corresponding to one isentrope the saturated state will be reached at the same pressure and temperature. From Eq. (6) it follows that for such jets the pressure p and collision frequency f will be the same for any given temperature interval (T, T − dT). Therefore, the number of collisions dz, just like the time interval dt, will depend only on the temperature in the forechamber for isentropic variation of the stagnation parameters. This dependence is reflected by the function

$$F(T, T_0) = (M^2 - 1)[u_*/(uT_*)]^{0.5} u^{-3} \exp(J_*/2R), \tag{12}$$

which is separated from the right side of Eq. (11). Its behavior is presented in Fig. 1 for stream temperatures T = 50, 100, 200, and 400°K (curves 1-4, respectively). It is seen that with an increase in the stagnation temperature the function F grows, and hence the time interval dt also increases. A comparison with the experiments of [1] shows that in this case the balance of bimolecular reactions of cluster growth and reactions of spontaneous decay results in an increase in cluster size, so that the isentropic generalization of the stagnation parameters yields the upper limit of the given average cluster size.

The lower limit can be obtained by requiring that variation of the stagnation conditions such that the number of collisions dz is constant. Let us realize this program.

For the region of stagnation temperatures of practical interest, from 200 to 2500°K, the function $F(T, T_0)$ can be approximated by the expression

$$F(T, T_0) \approx b(T) T_0 \exp(a_T T_0),$$

where $a_T = 8.54 \cdot 10^{-4}$ °K^{-1}.

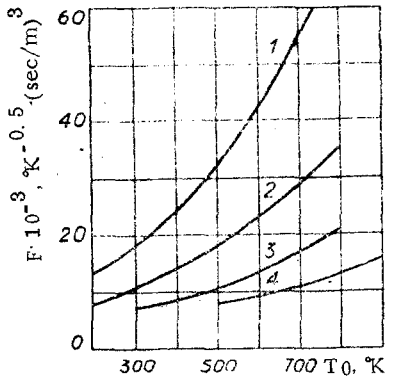

Fig. 1

Then for dz we have

$$dz \sim p(T, A_s)T_0 \exp(a_T T_0).$$

The condition of constancy of dz for a given temperature interval (T, T − dT) leads to the following relation between the stagnation pressure and temperature:

$$\ln p_0 = J_0/R - \ln T_0 - a_T T_0 + c_z, \tag{13}$$

where c_Z is a constant. As one moves along this curve toward an increase in stagnation temperature the contribution of bimolecular reactions to cluster growth will not change, whereas the contribution of spontaneous evaporation will grow due to the increase in the time interval dt. Thus, the average cluster size will decrease.

We have obtained the upper and lower limiting functions generalizing the stagnation conditons under which the average cluster size remains constant. They are expressed by Eqs. (6) and (13). It can be expected that the best approximation to the true function will be the average

$$\ln p_0 = J_0/R - 0.5 \ln T_0 - 0.5 a_T T_0 + c_N, \tag{14}$$

where c_N is a constant.

Experimental results taken from [8] are presented in Fig. 2. They correspond to stagnation conditions yielding a so-called average characteristic cluster size (N/z)* of 1000 in the far field of flow (the true size N of a neutral cluster is three to five times larger than the characteristic size [8]). The data are reduced to the equivalent diameter $d = 2.4 \cdot 10^{-3}$ m of the sonic nozzle [1]. The solid line in Fig. 2 is an approximation based on Eq. (14). The dashed lines 1 and 2 are the limiting curves corresponding to Eqs. (13) and (6). As we see, Eq. (14) well describes the experimental data. Approximate similarity laws for the transition from one set of initial conditions to another in the condensation of a given substance were formulated in [9]. They come down to the requirement of conservation of the number of collisions which a vapor molecule undergoes in the entire time from the instant of saturation to infinity. It is interesting to check whether this condition is satisfied for our case. For this purpose we choose two points on the main curve of Fig. 2 sufficiently far apart, with stagnation temperatures $T_{01} = 200°K$ and $T_{02} = 500°K$, and calculate the corresponding numbers of collisions for them by integrating Eq. (11). Their ratio proves to equal $z_2 : z_1 = 1.24$ (the agreement obviously will be better for closer points). Thus, the similarity law which we found in the form (14) satisfies the condition of conservation of the number of binary collisions beyond the phase-transition curve.

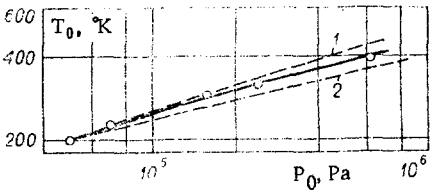

Fig. 2

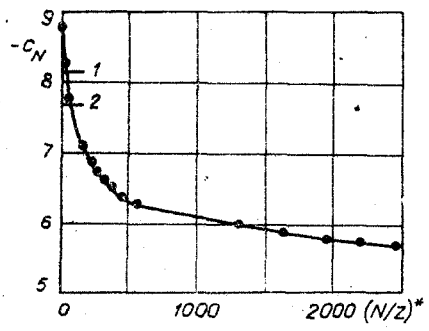

Fig. 3

In [4] to describe the condensation of expanding steam with the initial conditions T_0 = 3390°K, $p_0 = 3.2 \cdot 10^6$ Pa, and d = 0.93 m, Eq. (1) was used with $\gamma = 1.33$ (i.e., without allowance for vibrational heat capacity). In the case with CO_2 this is approximately equivalent to using $\gamma = 1.4$ for a temperature T_0 = 3000°K, whereas drawing a tangent to the curve (14) and determining γ from Eq. (1) yields $\gamma = 1.168$.

For a given nozzle geometry, a certain value of the constant c_N corresponds to each given average cluster size, according to Eq. (14). An experimental dependence of the average characteristic cluster size [$(N/z)^* \geqslant 150$] on the stagnation pressure for a temperature T_0 = 298°K in the forechamber was obtained in [10]. In [11], where the method of laser diagnostics was used, a similar dependence was obtained for a cluster size down to unity. In Fig. 3 the data of both reports are presented in the form of the dependence of the parameter c_N on the quantity $(N/z)^*$ (the experimental data used are denoted by points on the curve). The data are reduced to a nozzle diameter $d = 8.1 \cdot 10^{-4}$ m. (The similarity law $p_0 d^{0.6}$ = const [1] is used). It is seen from Fig. 3 that with an increase in average cluster size the corresponding stagnation conditions at first vary rapidly for small values of $(N/z)^*$, but in the region of $(N/z)^* \geqslant 500$ small changes in the stagnation conditions correspond to considerable changes in average cluster size. This indicates that condensation processes proceed very actively in this region. In molecular-beam investigations of condensation in CO_2 [3] in the region of stagnation temperatures from 234 to 300°K values of the stagnation pressure were obtained at which a minimum is observed in the molecular-beam intensity. Treating these data in accordance with Eq. (14) yields the value of $c_N = -8.15$ marked by the horizontal line 1 in Fig. 3. The line 2 ($c_N = -7.69$) corresponds to the experiments of [12], where stagnation conditions were obtained for which a maximum of the dimer signal is observed. It is known [3] that the minimum molecular-beam intensity corresponds to an earlier stage of condensation. In accordance with this, it follows from Fig. 3 that the minimum molecular-beam intensity corresponds to a smaller average cluster size than that for the maximum dimer signal.

The possibility of a comparison with one or another effects in condensing jets at a certain stage of condensation permits a unified interpretation of results in terms of the average cluster size. Here one must keep in mind that under certain stagnation conditions the observed phenomena may be subject to the influence of viscosity and nonequilibrium effects. The correlation of stagnation parameters through Eq. (14) is also a criterion for how strongly the observed phenomenon is connected with the condensation process.

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STABILITY OF PLANE-PARALLEL ELECTROHYDRODYNAMIC FLOWS IN A LONGITUDINAL ELECTRIC FIELD

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In recent years there have been significant developments in a new branch of electromagnetic hydrodynamics, electrohydrodynamics. Theoretical studies in this field have been stimulated by various technological applications [1-5]. Functioning electrohydrodynamic pumps, generators, and dc transformers have now been constructed. However, further development of electrohydrodynamic (EHD) devices is being hindered by the incomplete, and sometimes, even contradictory, nature of existing concepts involving the principles of electrophysics and the hydromechanics of weakly conducting liquids and gases [2]. One may say that the least-studied area involves questions of stability and turbulence of EHD flows. The present study will analyze the stability of plane-parallel to EHD flows in a longitudinal electric field with respect to small perturbations.

We choose as a characteristic length the quantity l_0, equal to one-half the channel width, and select as the characteristic velocity V_0. Let E_0 be the intensity of the externally applied electric field. We will measure electric field intensity, space charge density, time, pressure, and current density in units E_0, ρ_e, l_0/V_0, ρV_0^2, $K\tilde{\rho}_e$, E_0 (where ρ is the liquid density and K is the ion mobility coefficient). Then the system of equations describing the EHD of a viscous incompressible fluid can be written in dimensionless form as

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -\nabla p + (1/\mathrm{Re})\Delta \mathbf{v} + \mathrm{Eu}_a \rho_e \mathbf{E} + \Gamma \nabla \mathbf{E}^2; \quad (1)$$

$$\operatorname{div} \mathbf{v} = 0; \quad (2)$$

$$\operatorname{rot} \mathbf{E} = 0; \quad (3)$$

$$\operatorname{div} \mathbf{E} = \mathrm{Re}_a \rho_e; \quad (4)$$

$$\partial \rho_e/\partial t + (1/\mathrm{M}_a) \operatorname{div} \mathbf{j} = 0; \quad (5)$$

$$\mathbf{j} = \rho_e(\mathrm{M}_a \mathbf{v} + \mathbf{E}) - (1/\mathrm{Re}_i)\nabla \rho_e. \quad (6)$$

Here $\mathbf{v}$ is velocity; $\mathbf{E}$, electric field intensity; ρ_e, space charge density; $\mathbf{j}$, current density; Re, Reynolds number; $\mathrm{Re}_a = \tilde{\rho}_e l_0/\varepsilon\varepsilon_0 E_0$, electrical Reynolds number; $\mathrm{M}_a = V_0/KE_0$, electric Mach number; $\mathrm{Re}_i = Kl_0E_0/D$, ionic Reynolds number; $\mathrm{Eu}_a = \tilde{\rho}_e E_0 l_0/\rho V_0^2$, electrical Euler number; $\Gamma = (\varepsilon - \varepsilon_0)E_0^2/2\rho V_0^2$, electrical pressure number (where ε is the dielectric permittivity of the fluid and D is the diffusion coefficient).

The Navier–Stokes equation (1) considers the effect of the electric field on the charged liquid (the term $\mathrm{Eu}_a \rho \mathbf{E}$) and the force acting on the weakly polarized dielectric in an

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